9-amaliy mashg'uloti

O'zgarmas koeffitsientli yuqori tartibli chiziqli differensial tenglamalarni yechish. Eyler tenglamasi. O'zgaruvchi koeffitsientli chiziqli differensial tenglamalarni integrallash usullari

1. O'zgarmas koeffitsientli chiziqli tenglamalar

n-tartibli tenglama:

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y' + a_0 y = 0$$

Yechish usuli:

- Qaralayotgan tenglama xarakteristik tenglama yordamida yechiladi:

$$a_n r^n + a_{n-1} r^{n-1} + \cdots + a_0 = 0$$

Ilk yechimlar xarakteristik tenglama ildizlariga qarab quyidagicha topiladi:

- Oddiy ildizlar uchun: $y_i = e^{r_i x}$
- Ko'p martalik ildizlar uchun: $y_i = x^k e^{r_i x}$
- Kompleks ildizlar uchun: $y_i = e^{\alpha x}(\cos \beta x, \sin \beta x)$

O'zgartirish**:**

$$x = e^t \quad \Rightarrow \quad y(x) = u(t)$$

Natijada: o'zgarmas koeffitsientli tenglamaga keladi. Shundan keyin yechim avvalgi usul bilan topiladi.

3. O'zgaruvchi koeffitsientli chiziqli tenglamalarni integrallash

Umumiy ko'rinish:

$$y^{(n)} + p_{n-1}(x) y^{(n-1)} + \cdots + p_0(x) y = f(x)$$

Yechish usullari:

- To'liq yechim, umumiy yechim va xususiy yechim
- Xususiy yechim topish usullari**:**

- Variatsiya konstantalar usuli
- Operator usuli (pastki tartibli tenglamalar uchun)
- Ko‘rsatkich usuli (agar yechimning ko‘rinishi taxmin qilinsa)

Amaliy misollar

Misol 1:

$$y'' - 3y' + 2y = 0$$

Yechish**:**
Xarakteristik tenglama:

$$r^2 - 3r + 2 = 0 \Rightarrow r_1 = 1, \quad r_2 = 2$$

Yechim,

$$y(x) = C_1 e^x + C_2 e^{2x}$$

Misol 2**:** Eyler tenglamasi

$$x^2 y'' - 3xy' + 4y = 0$$

O‘zgartirish**:**

$$x = e^t, y(x) = u(t)$$

$$x = e^t \Rightarrow \frac{dy}{dx} = \frac{du}{dt} \cdot \frac{dt}{dx} = \frac{1}{x} \frac{du}{dt}$$

Natijada:

$$u'' - 4u' + 4u = 0 \Rightarrow (r-2)^2 = 0 \Rightarrow r = 2(ikkimarta)$$

Yechim:

$$u(t) = (C_1 + C_2 t)e^{2t} \Rightarrow y(x) = (C_1 + C_2 \ln x)x^2$$

Misol 3**:** O‘zgaruvchi koeffitsientli tenglama

$$xy'' + y' = 0$$

Bo‘lsin y′=z, demak:

$$xz' + z = 0 \Rightarrow z' + \frac{1}{x}z = 0$$

Ajratilgan o‘zgaruvchilar usuli:

$$\frac{dz}{z} = -\frac{1}{x}dx \Rightarrow \ln|z| = -\ln|x| + C \Rightarrow z = \frac{C_1}{x}$$

Endi

$$y' = \frac{C_1}{x} \Rightarrow y = C_1 \ln x + C_2$$

Mustaqil bajarish uchun topshiriqlar

1. Tenglamani yeching:

$$y''' - 6y'' + 11y' - 6y = 0$$

2. Eyler tenglamasini yeching:

$$x^2y'' + xy' - 9y = 0$$

3. Quyidagi tenglamani integrallang:

$$x^2y'' + 3xy' + y = 0$$

Nazorat savollari

1. O‘zgarmas koeffitsientli tenglamalarni yechish uchun qanday usuldan foydalaniladi?
2. Eyler tenglamasining umumiy ko‘rinishi qanday?
3. O‘zgaruvchi koeffitsientli tenglamani integrallashda qanday usullar mavjud?