13-mavzu amaliy mashg’uloti

O‘zgarmas koeffitsientli differensial tenglamalar sistemasi va uni yechish usullari

1. Differensial tenglamalar sistemasi — bu bir nechta noma'lum funksiyalar va ularning hosilalari ishtirok etadigan bir vaqtning o‘zida yechiladigan tenglamalar to‘plami.

O‘zgarmas koeffitsientli bo‘lishi — sistemadagi barcha koeffitsientlar o‘zgarmas sonlar bo‘lishi.
Bir jinsli bo‘lmagan bo‘lishi — tenglamada mustaqil funksiyalar (masalan, , $\sin t$, e^t, $\cos t$,va hokazo) mavjud bo‘lishi.
2. Umumiy ko‘rinishi

Ikki noma'lum funktsiyali sistema:

$$\begin{cases} x'(t) = a_{11}x + a_{12}y + f_1(t) \\ y'(t) = a_{21}x + a_{22}y + f_2(t) \end{cases}$$

yoki matritsa shaklida:

$$\mathbf{X}'(t) = A\mathbf{X}(t) + \mathbf{F}(t)$$

bu yerda:

$\mathbf{X}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$,
$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ — o'zgarmas koeffitsientli matritsa,
$\mathbf{F}(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$ — bir jinsli bo'lmagan a'zolar vektori

Yechish usullari

3.1. Bir jinsli sistemaning yechimi (F(t)=0)

Tenglama:

$$\mathbf{X}' = A \cdot \mathbf{X}$$

Yechish bosqichlari:

1. Xarakteristik tenglama tuzish:
$$\det(A - \lambda I) = 0$$
2. O‘z qiymatlar λ_1, λ_2 ni topish.

3. Har bir λ_i uchun **o‘z vektor** $\mathbf{v}_i$ ni topish:
   $(A - \lambda_i I)\mathbf{v}_i = 0$
4. Umumiy yechim:

$$\mathbf{X}_{\text{umumiy}}(t) = C_1\mathbf{v}_1 e^{\lambda_1 t} + C_2\mathbf{v}_2 e^{\lambda_2 t}$$

3.2. Bir jinsli bo‘lmagan sistemaning yechimi

Tenglama:

$$\mathbf{X}' = A\mathbf{X} + \mathbf{F}(t)$$

Yechish bosqichlari:

1. Bir jinsli qismning yechimini (yuqoridagidek) topish: $\mathbf{X}_{\text{umumiy}}(t)$
2. Xususiy yechim $\mathbf{X}_{\text{xus}}(t)$ topish:

F(t) ning shakliga qarab tanlanadi.
Tajriba usuli Masalan, agar

$$\mathbf{F}(t) = \begin{bmatrix} e^t \\ 0 \end{bmatrix}$$

tajriba:

$$\mathbf{X}_{\text{xus}}(t) = \begin{bmatrix} a \\ b \end{bmatrix} te^t$$

3. To‘liq yechim:

$$\mathbf{X}(t) = \mathbf{X}_{\text{umumiy}}(t) + \mathbf{X}_{\text{xus}}(t)$$

4. Amaliy misol

Berilgan sistema:

$$\begin{cases} x' = 2x + y + e^t \\ y' = 3x + 4y \end{cases}$$

1.Xarakteristik tenglama:

$$\lambda^2 - 6\lambda + 5 = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = 5$$

2.O‘z vektorlar:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

3.Bir jinsli yechim:

$$\mathbf{X}_{\text{umumiy}}(t) = C_1\mathbf{v}_1 e^t + C_2\mathbf{v}_2 e^{5t}$$

4.Xususiy yechim:

$$\mathbf{X}_{\text{xus}}(t) = \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} te^t$$

5.To‘liq yechim:

$$x(t) = C_1e^t + C_2e^{5t} - \frac{1}{2}te^t, \quad y(t) = -C_1e^t + 3C_2e^{5t} + \frac{3}{2}te^t$$

Quyidagi misolni ham qaraylik

$$\begin{cases} x' = x + 2y + 2t^2e^t \\ y' = 2x + 2y - te^t \end{cases}$$

Xususiy yechimini topaylik

1-bosqich: Xususiy yechim ko‘rinishini tanlash

O‘ng tomonda:

- birinchi tenglamada: $2t^2e^t$
- ikkinchi tenglamada: $-te^t$

Bu t^ne^t ko‘rinishda bo‘lganligi uchun **yechim** quyidagicha olinadi:

$$x(t) = (a_2t^2 + a_1t + a_0)e^t$$
$$y(t) = (b_2t^2 + b_1t + b_0)e^t$$

Bu yerda

$$a_0, a_1, a_2, b_0, b_1, b_2$$

noma'lum o'zgarmaslar (aniqlanadi).

2-bosqich: Hosilalarni topamiz

Quyidagi formulalarni esga olamiz (zanjirli hosila):

$$x' = \left[(a_2t^2 + a_1t + a_0)e^t\right]' = (2a_2t + a_1)\,e^t + (a_2t^2 + a_1t + a_0)e^t$$

Shunga o‘xshab:

$$x' = \left(2a_2t + a_1 + a_2t^2 + a_1t + a_0\right)e^t$$

$$x' = (a_2t^2 + (2a_2 + a_1)t + a_1 + a_0)e^t$$

Xuddi shunday:

$$y' = (b_2t^2 + (2b_2 + b_1)t + b_1 + b_0)e^t$$

3-bosqich: Sistemaga qo‘yib, koeffitsientlarni solishtiramiz

1-tenglama:

$$x' = x + 2y + 2t^2e^t$$

Tajriba yechimni qo‘yamiz:

Chap tomon:

$$x' = (a_2t^2 + (2a_2 + a_1)t + a_1 + a_0)e^t$$

O‘ng tomon:

$$x + 2y + 2t^2e^t = [(a_2t^2 + a_1t + a_0) + 2(b_2t^2 + b_1t + b_0)]e^t + 2t^2e^t$$

$$= (a_2 + 2b_2 + 2)t^2 + (a_1 + 2b_1)t + (a_0 + 2b_0) \cdot e^t$$

Endi **tenglashtiramiz:**

$$a_2 = a_2 + 2b_2 + 2 \Rightarrow 2b_2 + 2 = 0 \Rightarrow b_2 = -1$$

$$2a_2 + a_1 = a_1 + 2b_1 \Rightarrow 2a_2 = 2b_1 \Rightarrow b_1 = a_2$$

$$a_1 + a_0 = a_0 + 2b_0 \Rightarrow a_1 = 2b_0$$

2-tenglama:

$$y' = 2x + 2y - te^t$$

Chap tomon:

$$y' = (b_2t^2 + (2b_2 + b_1)t + b_1 + b_0)e^t$$

O‘ng tomon:

$$2x + 2y - te^t = 2[(a_2t^2 + a_1t + a_0) + (b_2t^2 + b_1t + b_0)]e^t - te^t$$

$$= (2a_2 + 2b_2)t^2 + (2a_1 + 2b_1 - 1)t + (2a_0 + 2b_0)$$

Tenglashtiramiz:

$$b_2 = 2a_2 + 2b_2 \Rightarrow 2a_2 = 0 \Rightarrow a_2 = 0$$
$$2b_2 + b_1 = 2a_1 + 2b_1 - 1 \Rightarrow 2(-1) + b_1 = 2a_1 + 2b_1 - 1 \Rightarrow$$
$$\Rightarrow b_1 = 2a_1 + b_1 + 1 \Rightarrow 0 = 2a_1 + 1$$

Yuqoridagilardan:

$$a_2 = 0, \quad a_1 = -\frac{1}{2}, \quad b_2 = -1, \quad b_1 = 0, \quad a_0 = (\text{erkin}), \quad b_0 = -\frac{1}{4}$$

Yechimi quyidagicha bo‘ladi

$$x(t) = \left(-\frac{1}{2}t + a_0\right)e^t, \quad y(t) = \left(-t^2 - \frac{1}{4}\right)e^t$$

Misolar

1. Quyidagi differensial tenglamalar sistemasini matritsa ko‘rinishida yozing:

$$\begin{cases} x' = 3x + 4y \\ y' = 2x + y \end{cases}$$

2. Quyidagi sistemaning xarakteristik tenglamasini tuzing va o‘z qiymatlarini toping:

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

3. Quyidagi sistema bir jinsli yoki bir jinsli bo‘lmagan ekanligini aniqlang:

$$\begin{cases} x' = x + y + e^t \\ y' = 2x - y \end{cases}$$

4. Quyidagi sistemaning umumiy yechimini toping:

$$\begin{cases} x' = 2x + y \\ y' = 3x + 4y \end{cases}$$

5. Quyidagi bir jinsli bo‘lmagan sistemaning xususiy yechim ko‘rinishini yozing:

$$\begin{cases} x' = x + 2y + \cos t \\ y' = -x + y + \sin t \end{cases}$$

6. Quyidagi sistemani to‘liq yeching (bir jinsli + xususiy yechim bilan):

$$\begin{cases} x' = x + 2y + e^t \\ y' = 2x + 2y - 3e^t \end{cases}$$