4-mavzu amaliy mashg’uloti

To'la differensial tenglamalar. Integrallovchi ko‘paytuvchini topish

To'la differensial tenglamalar.

Ta’rif. Agar

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

ko‘rinishdagi tenglamaning chap qismi biror u(x,y) funksiyaning to‘la differensiali, ya’ni

$$du = M(x,y)\,dx + N(x,y)\,dy \qquad (1)$$

bo‘lsa, u holda bunday tenglama to‘la differensialli tenglama deyiladi.

(1)tenglama to‘la differensialli tenglama bo‘lishi uchun quyidagi shart bajarilishi kerak:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

To‘la differensialli tenglama ta’rifidan du=0, bundan u(x,y)=C ekani kelib chiqadi (bu yerda C – ixtiyoriy o‘zgarmas).

u(x,y) ni topish uchun y ni o‘zgarmas deb hisoblaymiz. U holda du=0 ekandan,

$$du = M(x,y)\,dx$$

bo‘ladi. Bu tenglikni x bo‘yicha integrallasak,

$$u = \int M(x,y)\,dx + \varphi(y) \qquad (2)$$

hosil bo‘ladi.

(2) tenglikni y bo‘yicha differensiallaymiz va natijani N(x,y) ga tenglaymiz, chunki

$$\frac{\partial u}{\partial y} = N(x,y)$$

Shunday qilib:

$$\int \frac{\partial M}{\partial y}\,dx + \varphi'(y) = N(x,y)$$

yoki:

$$\varphi'(y) = N(x,y) - \int \frac{\partial M}{\partial y}\,dx \quad (3)$$

(3) ifodani y bo‘yicha integrallab, $\varphi(y)$ ni topamiz:

$$\varphi(y) = \int \left[N(x,y) - \int \frac{\partial M}{\partial y}\,dx \right] dy + C$$

Shunday qilib, umumiy yechim:

$$u(x,y) = \int M(x,y)\,dx + \int \left[N(x,y) - \int \frac{\partial M}{\partial y}\,dx \right] dy + C$$

Bu ifodani ixtiyoriy o‘zgarmasga tenglab, tenglamaning umumiy integralini hosil qilamiz.

1-misol.

$$(3x^2 + 6xy^2)\,dx + (6x^2y + 4y^3)\,dy = 0$$

tenglamaning umumiy yechimini toping.

Yechish:

Bu yerda:

- $M(x,y) = 3x^2 + 6xy^2$
- $N(x,y) = 6x^2y + 4y^3$

To‘lalikni tekshiramiz:

$$\frac{\partial N}{\partial x} = 12xy, \quad \frac{\partial M}{\partial y} = 12xy$$

demak,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

sharti bajariladi, ya’ni tenglama **to‘la differensial tenglama.**

$$\frac{\partial u}{\partial x} = M(x,y) = 3x^2 + 6xy^2$$

Bu tenglikni x bo‘yicha integrallaymiz:

$$u = \int (3x^2 + 6xy^2)\,dx = x^3 + 3x^2y^2 + \varphi(y)$$

Bundan:

$$\frac{\partial u}{\partial y} = 6x^2y + \varphi'(y)$$

Lekin,

$$\frac{\partial u}{\partial y} = N(x,y)$$

ekanligini hisobga olsak:

$$\varphi'(y) = 6x^2y + 4y^3 - 6x^2y = 4y^3$$

Shundan:

$$\varphi(y) = y^4 + C$$

Umumiy yechim:

$$u = x^3 + 3x^2y^2 + y^4 + C$$

yoki:

$$x^3 + 3x^2y^2 + y^4 = C$$

Integrallovchi ko‘paytuvchini topish.

Agar

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

bo‘lsa, u holda ba’zi bir shartlar bajarilganda, shunday $\mu(x, y)$ funksiyani topish mumkinki,

$$\mu M\, dx + \mu N\, dy = du$$

bo‘ladi. Bu $\mu(x, y)$ funksiya integrallovchi ko‘paytuvchi deyiladi.

Quyidagi hollarda integrallovchi ko‘paytuvchini topish oson:

1) Agar

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \Big/ N = \Phi(x)$$

bo‘lsa,

$$\ln \mu = \int \Phi(x)\, d$$

2) Agar

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \Big/ M = \Phi_1(y)$$

bo‘lsa

$$\ln \mu = \int \Phi_1(y)\, dy$$

2-misol. Tenglamani yeching.

$$(y + xy^2)\, dx - x\, dy = 0$$

Yechish. Bu yerda:

$$M = y + xy^2, \quad N = -x, \quad \frac{\partial M}{\partial y} = 1 + 2xy$$

$$\frac{\partial N}{\partial x} = -1$$

Shunday qilib:

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Demak, tenglamaning chap tomoni biror funksiyaning to‘la differensiali emas.

Endi, bu tenglamaning faqat y ga bog‘liq bo‘lgan integrallovchi ko‘paytuvchisi bormi, degan masalani ko‘rib chiqamiz.

Hisoblaymiz:

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \Big/ M = \frac{-1 - (1 + 2xy)}{y + xy^2} = \frac{-2(1 + xy)}{y + xy^2} = \frac{2}{y}$$

Shundan,

$$\ln \mu = -2 \ln y \quad \Rightarrow \quad \mu = \frac{1}{y^2}$$

Tenglamani μ ga ko‘paytiramiz:

$$\left(\frac{1}{y} + x\right) dx - \frac{x}{y^2} dy = 0$$

Hosil bo‘lgan tenglama **to‘la differensial tenglama,** chunki:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = -\frac{1}{y^2}$$

Tenglamani yechib, quyidagi ifodani hosil qilamiz:

$$\frac{x}{y} + \frac{x^2}{2} + C = 0$$

yoki undan y ni topamiz:

$$y = -\frac{2x}{x^2 + 2C}$$

Bu yechim — tenglamaning **umumiy integrali** hisoblanadi.

Quyidagi differensial tenglamalarning chap tomonlari to‘liq differensialdan iborat ekanligi tekshirilsin va tenglamalar yechilsin:

1) $(e^x + y + \sin y)\,dx + (e^y + x + x\cos y)\,dy = 0$

2) $(x + y - 1)\,dx + (e^y + x)\,dy = 0$

3) $(x\cos y - y\sin y)\,dy + (x\sin y + y\cos y)\,dx = 0$

4) $2xy\,dx + (x^2 - y^2)\,dy = 0$

5) $(2 - 9xy^2)\,dx + (4y^2 - 6x^3)\,dy = 0$

6) $\frac{y}{x}\,dx + (y^3 + \ln x)\,dy = 0$

Quyidagi differensial tenglamalarning integrallovchi ko‘paytuvchilari topilsin va tenglamalar yechilsin:

1) $(x^2 - y)\,dx + x\,dy = 0$
2) $y^2\,dx + (yx - 1)\,dy = 0$
3) $(x^2 + y^2 + x)\,dx + y\,dy = 0$
4) $xy^2(xy' + y) = 1$

5) $(x^2 + 3\ln y)\, y\, dx = x\, dy$

6) $2\,\mathrm{tg}\, y\, dx + (x^2 - 2\sin y)\, dy = 0$